Decentralized Primary Frequency Control in Power Networks

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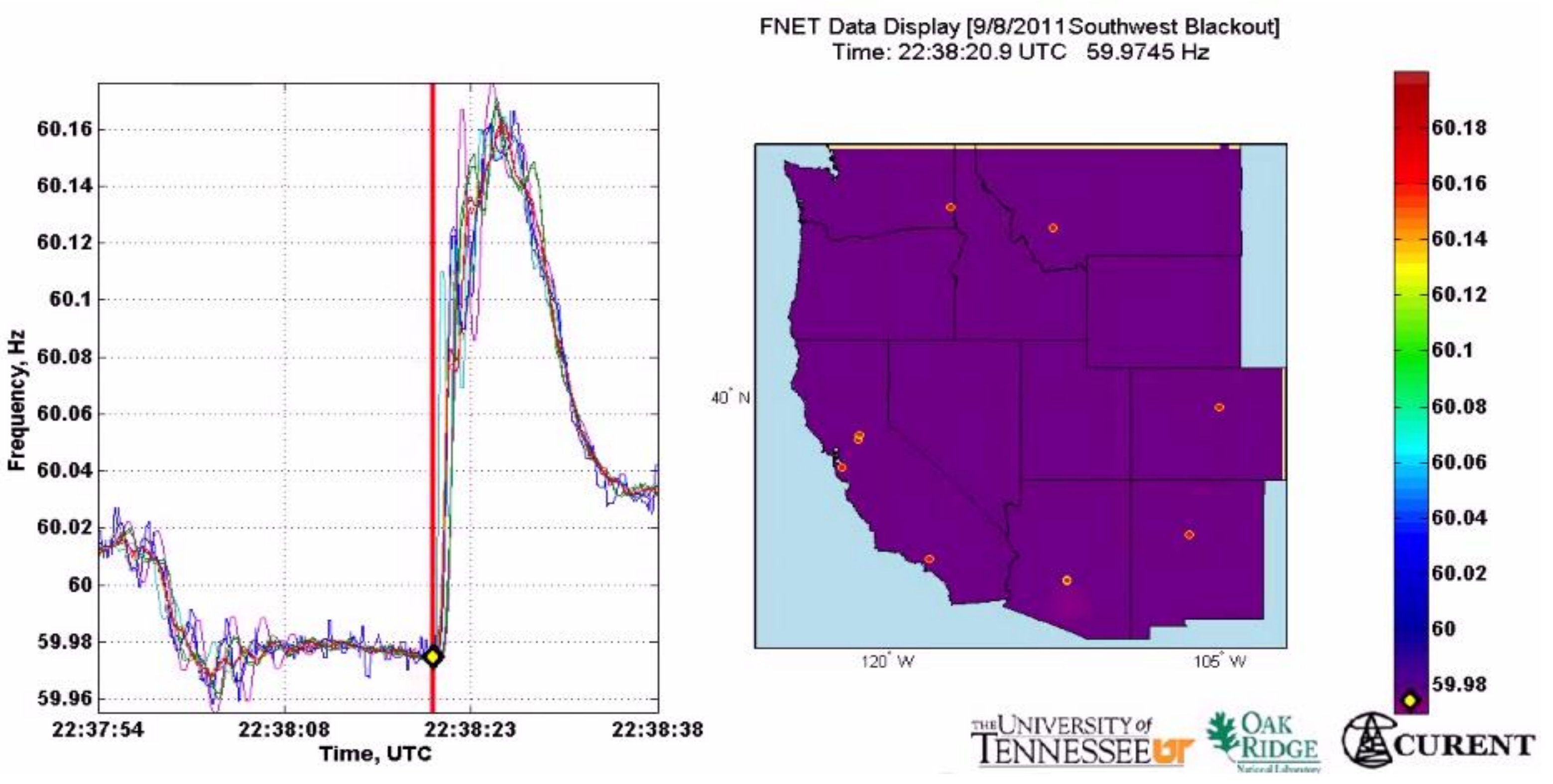
Motivation

- ❑ Normal operation of power networks: all buses synchronized to nominal frequency (60 Hz)
- ❑ **Supply-demand imbalance → frequency deviation**
  degrade load performance; overload transmission lines; trigger protection devices; damage equipment



WECC frequency profile, 9/8/2011 Southwest Blackout

Primary frequency control: balance power and stabilize frequency

- ❑ Traditionally done on generator side
- ❑ Increased power intermittency (33% renewable in CA by 2020) requires faster and more spinning reserves, which have higher cost and emission

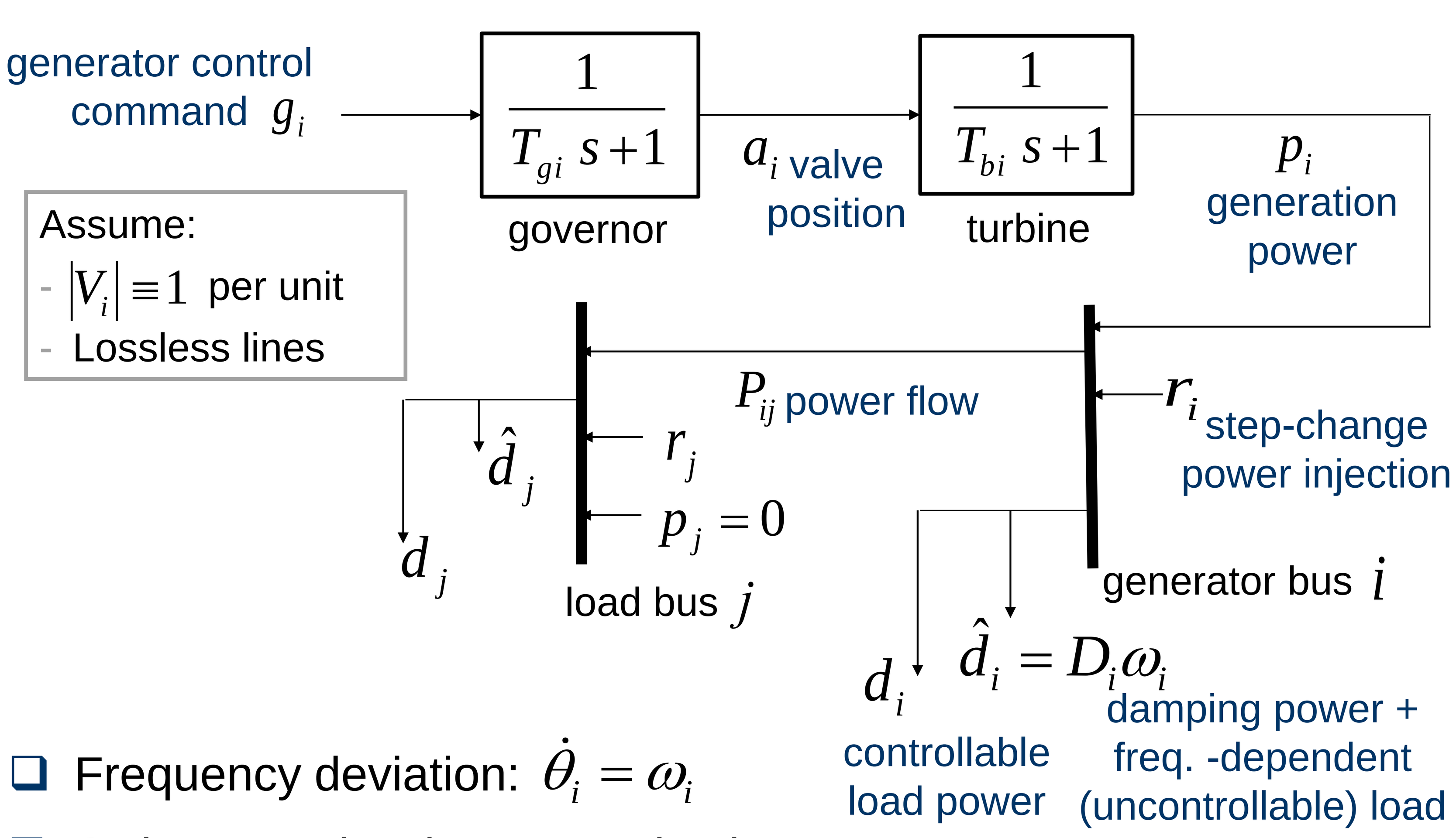
Load-side participation in primary frequency control

- ❑ A faster and cleaner supplement to generator-side
- ❑ Decentralized control for scalable and flexible plug-n-play
- ❑ Challenge: Joint design and stability analysis with generator-side (purpose of this work)

Main Results

- ❑ Design decentralized primary frequency control which operates jointly on generators and loads
- ❑ Stabilize bus frequencies and achieve economic efficiency at closed-loop equilibrium
- ❑ Prove asymptotic stability with a relatively realistic generator model and nonlinear AC power flows
- ❑ Show performance improvement with simulation

Power Network Model



- ❑ Frequency deviation: $\dot{\theta}_i = \omega_i$
- ❑ Swing equation (generator-bus):

$$M_i\dot{\omega}_i = r_i + p_i - d_i - \hat{d}_i - \sum_{j\in N(i)} P_{ij}$$

- ❑ Power balance (load-bus):

$$0 = r_i + p_i - d_i - \hat{d}_i - \sum_{j\in N(i)} P_{ij}$$

- ❑ AC power flow: $P_{ij} = B_{ij}\sin(\theta_i - \theta_j)$

Technical Approach

Problem Formulation

Design **decentralized** controllers for (g, d), such that the closed-loop system has an **asymptotically stable** equilibrium, where $(p^*, d^*, \hat{d}^*, P^*)$ solves the **economic efficiency** problem:

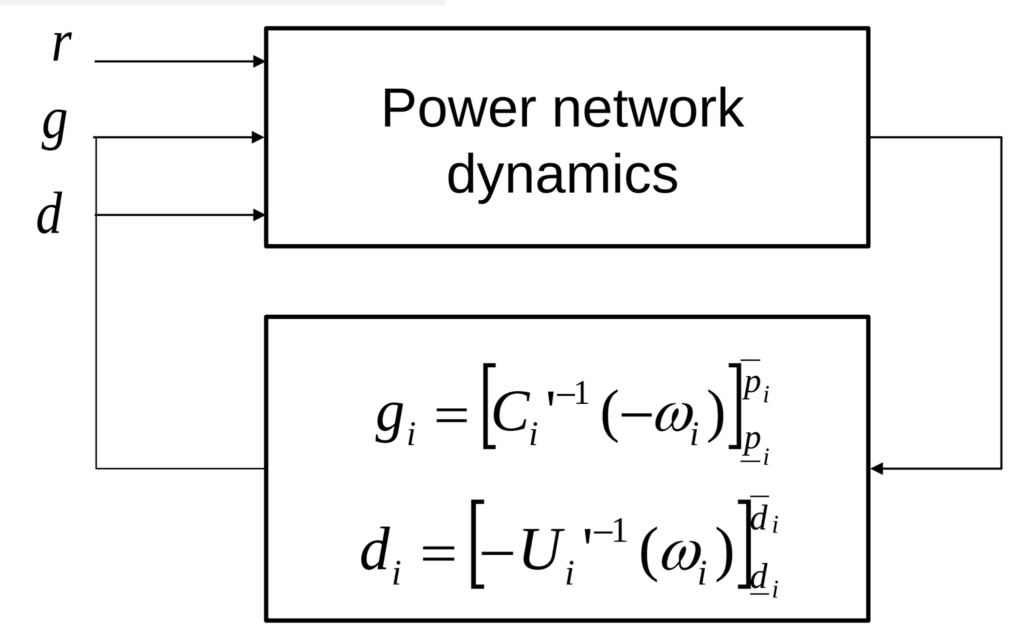
user utility (concave) generation cost (convex) penalty for freq. deviation

$$\max_{p,d,\hat{d},P} \quad \sum_i U_i(d_i) - C_i(p_i) - \frac{\hat{d}_i^2}{2D_i}$$

$$\text{s.t.} \quad r_i + p_i - d_i - \hat{d}_i - \sum_{j\in N(i)} P_{ij} = 0 \qquad \forall i$$

$$\underline{p}_i \le p_i \le \overline{p}_i, \qquad \underline{d}_i \le d_i \le \overline{d}_i \qquad \forall i$$

Controller Design



Equilibrium Analysis

- ❑ All closed-loop equilibria are economically efficient
- ❑ Dual optimal $\omega_i^* = \omega_j^*$: Bus frequencies stabilized to the same
  - Proof approach: Equilibrium condition ⇒ KKT condition
  - Need secondary frequency control to restore bus frequencies to the nominal value

Stability analysis

Lyapunov function candidate: $V = E + \sum_{i\in\text{gen}} V_i$

where $E = \frac{1}{2}\sum_{i\in\text{gen}} M_i\Delta\omega_i^2 + \sum_{(i,j)\in\text{line}} \int_{\theta_{ij}^*}^{\theta_{ij}} B_{ij}(\sin u - \sin\theta_{ij}^*)du$

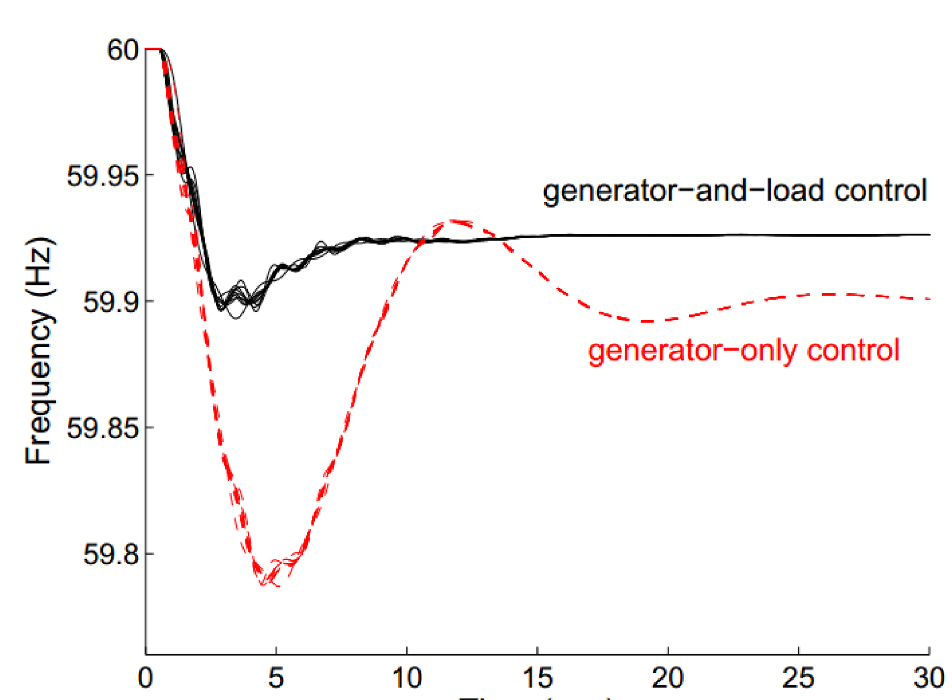
kinetic energy potential energy

and $V_i = [\Delta a_i, \Delta p_i]\, P_i\, [\Delta a_i, \Delta p_i]^T$ with $P_i \succ 0$

Construct P_i such that V is a Lyapunov function, proving asymptotic stability of any equilibrium satisfying mild conditions

Simulation Result

IEEE 39-bus test case, in Power System Toolbox (Chow et al.)



Show generator frequencies when:

- **Red**: only generators are controlled
- **Black**: 50% control capacity on generators and 50% on loads

Both cases: same total control capacity

Load-side participation improves both steady-state and transient

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